

The History of the Gothenburg–Reykjavík–Strathclyde Combinatorics Group

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ABSTRACT: This is an overview of the history of the combinatorics group that was born in Gothenburg, Sweden, in the late 1990s and then lived in Reykjavík, Iceland, and now in Glasgow, Scotland.

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1. Origins in Gothenburg

In the summer of 1990, while I was still a graduate student at the Massachusetts Institute of Technology, advised by Richard Stanley, I moved to Gothenburg, Sweden, for personal reasons*. I was lucky in that Richard visited Stockholm a couple of times during the following academic year, so I could meet him there. Once I finished my Ph.D. I got a job in 1992 at the joint mathematics institute of the Chalmers University of Technology and the University of Gothenburg. I was the only combinatorialist in this rather big department, dominated, as much of Swedish mathematics, by analysis, which was understandable, given all the strong analysts in Sweden throughout the 20th century. In fact, it looked uncertain for a while whether I would get a job there, but I later found out that one of the few algebraists in the department had supported my case very strongly, which may have been the deciding factor, since back then it was quite common to hire mostly local talent.

In 1998 I got a grant that allowed me to hire a Ph.D. student. As I recall, when I advertised I only got one application. But I was lucky; this was Anders Claesson. The following year one of the applicants for the department's general round of admissions was Sergey Kitaev. He had already done some work in combinatorics on words, and knew that he wanted to do combinatorics for his Ph.D., so he became my student. And then in 2000 Petter Brändén started his Ph.D. work with me. At that point, this already felt like an active research group, not just because of the common threads in our research, which at this point was mostly about permutation patterns and related objects, but also because we ran a [combinatorics seminar](#), which was very active. In addition to the four of us giving talks, we also had colleagues from the Computer Science department speak, as well as students from a seminar course in combinatorics I gave. We also had quite a few visitors coming through Gothenburg who gave talks in this seminar, as well as students doing undergraduate or master's theses in combinatorics.

At the risk of offending somebody (who I hope will not be reading this) I think it is safe to say that our combinatorics seminar was quite popular in the department, beyond our own group, because it was very lively, compared to what seemed to be the case for some other, established, seminars in the department. Namely, there was always much informal discussion, and speakers were frequently interrupted in the middle of a talk, asked to explain things better, or because somebody had a point to make about the subject. So, talks in our seminar were characterized by a natural and fruitful dialogue between speaker and audience, more in line with what I was used to in seminars I had attended in the US.

We also had reading group seminars on central problems in combinatorics, which were very popular among the graduate students. I suggested to Petter that he read up on and give a presentation on the Poset Conjecture, also known as the Neggers-Stanley conjecture. This topic turned out to be central to his thesis, and initiated his interest in zeros of polynomials in combinatorics.

As mentioned above, our research at this time was quite focused on permutation patterns, in particular the generalized patterns that Eric Babson and I had introduced in 2000 [4], after he visited us in Gothenburg.

*My sister won big in the lottery, paid for me to go to a combinatorics conference in Stockholm in 1989 (where, incidentally, I.M. Gelfand unexpectedly popped up and gave a short talk, on 24 papers of his as he explained it), which led to my tracking down a woman I had known 14 years earlier, and she lived in Gothenburg.

These patterns are now referred to as vincular patterns — a word coined by Anders, from the Latin *vinculum*, for bond. Anders, Sergey and myself have done much work on patterns since, but Petter soon took off on a different line of research, namely positivity problems in various branches of mathematics, in addition to his interest in algebraic combinatorics. Anders and Petter each visited other universities during their time as graduate students, Anders in Waterloo in the spring of 2002 where he coauthored a paper with Godsil and Wagner on a curious group defined in terms of any given poset and its lattice of order ideals [30], Petter in Rome 2003-4, with Francesco Brenti, whose work on properties of combinatorial polynomials no doubt played a part in Petter’s later substantial work in the area.

Before I had any students, I spent a total of 3 months in 1995/6 on a postdoc grant at the University of Strasbourg, invited by Dominique Foata, a great pioneer of algebraic combinatorics. This was funded by a network that was a predecessor to the *EU-Network in Algebraic Combinatorics* (ACE), which funded Petter’s visit to Rome mentioned above. There I started talking to Jiang Zeng, who was a local and is now in Lyon, and Bob Clarke, who was visiting from Adelaide. We wrote a paper on new and old permutation statistics and their relations to Motzkin paths and continued fractions [41] (and to work of several other authors on similar things). For some reason that I have now forgotten, this work led me to look at statistics on ordered set partitions, on which I wrote a paper a couple of years later. This paper was rejected by two journals and I then posted it on my website in 2001, but otherwise put it aside, until 2006, when I posted it on the arXiv, because some of its conjectures had been proved by Ishikawa, Kasraoui, and (Jiang) Zeng [66]. Then, in 2019, I was asked if I would consider submitting the paper, by somebody who thought it had been influential enough to deserve being published. This I did, and on this third try, more than two decades after the paper was written, it was not rejected [97].

One paper I wrote during this time was not supposed to have anything to do with permutations, although permutation statistics occupied my mind to a great extent already then. My goal had been to answer (in a modified form) a question of Brenti’s, whether for an arbitrary finite graph G there could exist a standard graded algebra whose Hilbert polynomial would equal the chromatic polynomial of G . In the end, however, permutations played a significant role in this paper [96], and they have managed to creep into most of my work, because they are often a convenient way to represent combinatorial structures.

In addition to my three Ph.D. students, we also had a visiting student from Waterloo for one semester, Antoine Vella, who authored two papers [79, 100] on patterns during his stay with us, one of them with two members of the group. We also had a postdoc in our group during this time, 2002-3. This was Toufik Mansour, who already then was as prolific as he has been ever since, authoring a number of papers while he was with us, among them [51, 74, 75, 78], several of these with other members of the group. Both Antoine and Toufik were funded through the ACE network, which we were by then a part of.

During our time in Gothenburg, Anders published the first paper on pattern avoidance for vincular patterns [26], and two sequels coauthored with Toufik [39, 40]. Sergey wrote his paper generalizing vincular patterns to having partially ordered entries instead of a total order [69] and quite a few others on patterns [67, 76, 77] (Toufik being a coauthor on two of these), while also publishing on combinatorics on words [54, 68]. Petter found counterexamples to the Neggers-Stanley conjecture [13] and introduced the class of sign-graded posets which generalizes the class of naturally labeled graded posets [15]. In [15] he also proved gamma-positivity for Eulerian polynomials of the posets in this class, which implies unimodality of their coefficients, which was one of the motivations for the Neggers-Stanley conjecture. He also developed a framework for proving real-rootedness of polynomials [14, 16].

2. Reykjavík years, 2005-2010

In 2005 I started working part-time at Reykjavík University (RU) in Iceland, splitting my time between there and Gothenburg. This quickly turned into a full-time job when I was asked to lead the development of mathematics teaching for much of the university. In particular, an entire new curriculum was to be developed for the Engineering School and the School of Education. Both of these were being started at RU at that point, after a merger with a technical college and an infusion of funds from the government to support that work and to start a new School of Health and Education.

Due to “political” differences within this largely new university, things turned out somewhat differently from what its then president had laid out. I do not want to go into much detail about the “power struggle” within RU, but it must be said that the combinatorics group that was built there had its home for most of its existence in the School of Health and Education. This may seem strange, in particular for a university with a strong emphasis on engineering and computer science. But, given the “political” atmosphere, this turned out to be for the best, while it lasted. Namely, the Dean of the School of Health and Education, Inga Dóra Sigfúsdóttir, with the support of the university president, wanted to build a program for current and future teachers of math in elementary and secondary schools, with a strong emphasis on training these teachers in math. The idea, which may seem natural to mathematicians while not exactly in vogue in education circles in Iceland (as in many

other countries), was that to be a good teacher of mathematics it was not enough to be trained in teaching in general, but that a strong grounding in the subject would be necessary. Crucially, our Dean was adamant that the academics to be hired to build this math education program be active researchers, and that we should hire the best we could get our hands on to build a strong and coherent research group, in combinatorics.

Thus I was charged, in the spring of 2005, with immediately hiring one other faculty member to start teaching in the fall, someone who was also to fit into the combinatorics group to be built. We got lots of applications from all over the world, several of them quite good, and ended up hiring Sergey, who had spent two years as a visiting assistant professor in the US, first at the University of Kentucky and then at the University of California, San Diego, and a semester at the Mittag-Leffler institute in Stockholm in between. Early the following year we got a so-called Excellence Grant from the Icelandic Research Fund that included a postdoc position for Anders, who in the meantime had been at Mittag-Leffler and at Kalmar University in Sweden, and he later got a faculty position at RU.

The three of us were now assembled again, and busy building this math education program, which quickly also led to an ordinary BSc program in math, since we had decided that the BEd program for future math teachers should contain the basic part of a BSc program in mathematics, in addition to the necessary educational part. We were also quite active in research, although I spent most of my time on the teaching programs and administration during the first few years. In 2006 we hosted the annual Permutation Patterns conference, which had started in Otago, New Zealand, three years earlier, and in 2010 the Nordic Combinatorial Conference.

Then, in 2007, Mark Dukes got a 3-year postdoc position at the other university in town, the University of Iceland. Mark, whom we first met at the Séminaire Lotharingien de Combinatoire in Domaine Saint Jacques in 2004, had a background in probability but was increasingly working in combinatorics and had been a postdoc in Rome with Francesco Brenti, under the auspices of the ACE network, which had funded so many visits of and with the group while in Gothenburg. The four of us, with Mark at the University of Iceland, now became the Reykjavík Combinatorics Group, and various research collaborations naturally arose between the members. One particularly noteworthy such collaboration, which also included Mireille Bousquet-Mélou, resulted in a very nice paper [12] on $(2+2)$ -free posets, ascent sequences, and permutations avoiding certain *bivincular* patterns. These bivincular patterns, whose introduction was the embryo out of which the paper grew, are a generalization of vincular patterns and they are closed under all the “trivial” symmetries on permutations, namely reverse, complement, and the group-theoretic inverse, the last of which vincular patterns lack. Anders and Petter then later generalized this further to the *mesh patterns* [17]. The paper [12] generated a lot of activity, with a great number of papers following up, several by members of the group [27, 28, 38, 44, 45, 47, 81], two of the above including one of our postdocs, Martina Kubitzke [28, 44], who also wrote a paper with Volkmar Welker on Veronese series of a rational power series and graded algebras [84].

Another very elegant paper worth mentioning is the classification by Anders and Sergey [33] of bijections between 321- and 132-avoiding permutations. This paper demonstrates — by analyzing which subsets of a set of over 20 permutation statistics are preserved under each bijection, in the sense of each statistic being mapped to another such — that several of the bijections in the literature were essentially the same, modulo some of the trivial symmetries on permutations.

In 2008, the group got to hire two postdocs at RU, funded by the university. We got plenty of good applications and ended up hiring Amy Glen and Robert Parviainen. Amy had done her Ph.D. at the University of Adelaide in Australia and then spent two years as a postdoc at LaCIM, Université du Québec in Canada. Robert had done his Ph.D. in Uppsala, Sweden, and was then for four years a Research Fellow at The University of Melbourne in Australia. After spending a year with us, Amy got a faculty position at Murdoch University in Perth, Australia, where she remained until 2021, when she started working for Apple, Australia. Robert went into industry in the US after two years as a postdoc with us, and is currently Senior VP Data Science at Seriously Digital Entertainment. While he was with us, Robert classified the bivincular patterns of lengths at most 3 according to Wilf-equivalence [86] and he coauthored with Mark a paper cited above [47] on certain matrices that are in bijection with ascent sequences.

In early 2009 we were awarded a second Grant for Excellence by the Icelandic Research Fund. On this grant we had Luca Zamboni, who was then at the University of North Texas, as co-principal investigator. He was to be hired to RU to join us, but this fell through because the Dean responsible dragged out on finalizing the contract until Luca had to accept or reject an offer from Université Lyon 1. He therefore accepted the position in Lyon, where he still remains. Another consequence of this was that Luca’s Ph.D. student Steve Widmer, who had been part of our group ended up receiving his Ph.D. [101] from Lyon, rather than RU. This time the Excellence grant was big enough for us to support several postdocs, and we hired Henning Úlfarsson, Vít Jelínek, Martina Kubitzke, and Anisse Kasraoui in 2009, and Chris Severs and Pavel Salimov in 2010. Of these, all stayed with us for one year, except for Henning, who was a postdoc under this grant until 2013, when he got a faculty position at RU. Now, Vít is at Charles University, Prague, Martina at Osnabrück University, Germany, Chris went to eBay after his postdoc and is currently at Benchling in Santa Cruz, California, and Pavel spent two years as a postdoc at the University of Liège, and is now at Google in Zürich.

During our time in Reykjavík Sergey started his pioneering work on word-representable graphs [63,80], a field

that has seen steady growth since then (more on that in the next section), and he also wrote a paper on self-dual maps [72] with de Mier and Noy. Another paper on planar maps, by Sergey, Pavel, Chris, and Henning [82] relates them to pattern avoiding permutations, and another three papers by Anders and Sergey, some also involving de Mier and myself [34–36], deal with the related $\beta(a, b)$ -trees. Several papers by members of the group, most including Anders and Mark, also Henning and some external coauthors, deal with sorting [2, 29, 37, 43, 99], but some of these and those on the $\beta(a, b)$ -trees were written after the move to Strathclyde.

In a paper of Sergey’s coauthored with Seif [83] they solved the word problem for the Perkins semigroup, which has played a central role in semigroup theory since 1960, particularly, as a source of examples and counterexamples. And in his paper with Ehrenborg and Perry [52] they used the spectral theory of integral operators on $L^2([0, 1]^m)$ to find asymptotics for the number of permutations avoiding a set of consecutive patterns.

A paper by Anders and Mark, with Chung and Graham [25], used juggling sequences and bubble sort to obtain formulas for the numbers of permutations with a given number of descents and maximum difference between a letter and its place in the permutation, thus mixing descents and excedances, which generally are hard to keep track of simultaneously.

Although Luca never got hired by RU he was still a co-PI on our Excellence Grant, and as such belonged formally to the group, even after he took up his position in Lyon. However, I only mention here some of his great number of papers during this period, since his collaboration with other group members was seriously diminished after he went to Lyon. Much of Luca’s work during this period was on combinatorics on words, and some of that with Amy and Steve [18, 19, 21, 55, 56, 60, 88, 89], whereas Amy also wrote quite a few papers with other members of the group and various other coauthors [3, 57–59, 61].

The move from RU to Strathclyde was precipitated by rather dramatic events that need to be explained briefly here. In the summer of 2010 I was summarily fired from RU, at the demand of the then president of the university, who had taken over early that year (incidentally, this was the former Dean who had derailed Luca’s hiring). The official reason given was necessary cutbacks because of the financial crisis of 2008, which had an outsize impact on Iceland, because of the gargantuan bankruptcies of its three big banks. It was plain to see, though, that this was not the reason, but rather my criticism of how the university was being run by its leadership, contrary to the advice of the international advisory board that this same leadership had assembled[†]. Ironically, one of the things this advisory board had said was that to compete internationally RU needed to have some kind of tenure system, offering long term job security for academics. There was no such system in place, so firing academics was easy.

This of course jeopardized the future of the group, not least because I was PI on the grant that supported all the postdocs, and which was supposed to last for another two years. For a couple of months things looked very uncertain, and protest letters in support of the group, from two groups of mathematicians from all over the world, apparently did not move the RU president to reconsider his decision. Then, in early August, when Sergey, Anders, Mark, and I were attending the FPSAC in San Francisco, I was contacted by one of the members of RU’s advisory board, who was Head of Department of Computer and Information Sciences at the University of Strathclyde in Glasgow. Ironically, she had been asked to join the advisory board by the RU president who had fired me and had tried in vain to get him to reverse that decision. She had then convinced the principal of Strathclyde, who had the means for such strategic moves, to consider hiring the entire group (Anders, Mark, Sergey, and myself). This of course went through a rigorous process of evaluations and interviews, but in the end that was the outcome; all four of us were offered positions at Strathclyde, and all accepted, moving in early 2011.

3. At Strathclyde, from 2011

In 2008 it was decided that the FPSAC conference (Formal Power Series and Algebraic Combinatorics) was to be held in Reykjavík in 2011, organized by our group. Although we had all moved to Strathclyde before the summer, we went through with this and the conference was held in Reykjavík in early June, with around 200 participants. The conference was dedicated to the memory of the great Philippe Flajolet, who had died in March of that year. The following year, Permutation Patterns 2012 was hosted by the group in Glasgow. Then, in 2016, we were privileged to organize the 26th British Combinatorial Conference. More recently, the Permutation Patterns 2021 Virtual Workshop was hosted online from Strathclyde.

Soon after moving to Strathclyde we got two Ph.D. students who both completed in 2015, Stuart Hannah, who worked on interval orders [31] and went into industry after his Ph.D., and Jason P. Smith who later was a postdoc in the group then spent two years at the University of Aberdeen and has since been at Nottingham

[†]In the wake of this, two members of the board resigned, and a third wrote a piece of scathing criticism of this violation of academic freedom, see <https://www.tc.columbia.edu/articles/2010/august/in-opinion-piece-tcs-allegante-defends-academic-freedom/>.

Trent University. Jason and Thomas Selig, who was also a postdoc with us, both in 2015-18, were supported by grants won by members of the group from the EPSRC, the UK's main fund for research in engineering and science. Thomas then spent a year as a postdoc at the University of Iceland and subsequently secured a position at Xi'an Jiaotong-Liverpool University in Suzhou, China. Jason's work, both during his Ph.D. and postdoc, focused mostly on the Möbius function and topology of combinatorial posets [91–95], in particular the poset of permutations ordered by pattern containment. The first paper on the topology of the permutation pattern poset was the result of a collaboration of mine with Peter McNamara [85]. Another paper that led me to suggest the Möbius function of the pattern poset to Jason I wrote while in Reykjavík, with Vít Jelínek, Eva Jelínková and Alex Burstein [20], Alex having started on this independently, then told me about his results, so we joined forces.

The focus of Thomas's work, in some of which Jason also participated, was on the Abelian Sandpile Model for certain graphs, and its relations to some other combinatorial structures [48–50, 90]. Both Mark and I also collaborated on this work, which was inspired by a paper of Mark's with Le Borgne [46] and a somewhat surprising connection to a paper of Lauren Williams and myself [98]. In fact, I had a few times, over many years, tried without any luck to find a bijection between the Le-tableaux in [98] (which were essentially present in a paper of Postnikov on total positivity and Grassmannians [87]) and the EW-tableaux essentially defined by Ehrenborg and van Willigenburg in their paper on Ferrers graphs [53]. When Thomas showed me a bijection between acyclic orientations with a unique sink on certain bipartite graphs, arising in [46], and permutations with excedances precisely in their first k places for some k , I realized this bijection I had been trying to find was staring me in the eye.

In 2016, both Anders and Mark left Strathclyde, Anders to take up a position at the University of Iceland, whereas Mark moved to University College Dublin. In the wake of that we got to hire a new combinatorialist, which resulted in David Bevan joining the group. David had recently completed his doctorate at The Open University, following a 25-year career as a software developer. His thesis concerned the growth of permutation classes, in which he determined the asymptotic growth rates of permutation grid classes [1, 6, 7], and established a new lower bound on the growth rate of the class of 1324-avoiding permutations [8] which he further improved in a later co-authored paper [10].

During our first year at Strathclyde Anders and I collaborated with Vít Jelínek on a paper motivated by trying to find bounds on the growth rate of the number of permutations avoiding the pattern 1324 [32]. This paper contains a conjecture (Conjecture 13, see also Conjecture 20) that is intriguing — because it is so simple and so “obviously” true — but has resisted all attempts so far. In fact, when Anders and I first started working on this we felt so certain this would be an easy lemma that we decided to leave its proof until last, thinking we had already bagged the result mentioned in Theorem 17, which depends on this conjecture being true. The current best bounds for this elusive growth rate appear in the paper by David and coauthors mentioned above [10], but I guess most of us now believe in the estimate of Conway, Guttman and Zinn-Justin [42].

Shortly after we moved to Strathclyde Sergey's survey book *Patterns in Permutations and Words* [70] appeared, and in 2015 his second book was published, *Words and Graphs* [73] coauthored with Vadim Lozin, surveying the theory of word-representable graphs, which Sergey initiated as mentioned before [71]. Since we moved to Glasgow Sergey has published extensively in both of these fields, but also papers on several other subjects, such as Riordan graphs [22–24, 65].

In 2019 Marc Glen completed his Ph.D. [62] and Kittitat Iamthong in 2021 [64], both working on word-representable graphs, advised by Sergey. Currently, David has two students: Noura Alshammari, who is looking at the enumeration of monotone grid classes of permutations, and Dan Threlfall, who is working on properties of random integer compositions and permutations, where David has been breaking new ground [5, 9]. Another paper with a probabilistic aspect, by Natasha Blitvić and myself, is [11], where numerous combinatorial sequences are shown to be moment sequences of probability measures on the real line, raising the question of whether there might be some interesting theory explaining when this is the case for combinatorially defined sequences.

In the spring of 2019, the leadership of the Department of Computer and Information Sciences, where we had been for eight years, decided that combinatorics did not belong in the department. For a while, it was unclear what the future of the group would be, but we received heart-warming reactions from the UK combinatorial community in support of the group, and Peter Cameron started a petition that received over five hundred signatures of mathematicians from all over the world (see here and here). In the end, it was decided that the group move to the Department of Mathematics and Statistics, where it has lived happily ever since.

In November 2021 I retired from Strathclyde, but I retain a position as a Research Professor, and so still belong to the group, although I will not have much to say about its future, which is now in the hands of Sergey and David.

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