ICECA
International Conference
Enumerative Combinatorics and Applications
University of Haifa – Virtual – August 17-19, 2026

STABILITY OF PLETHYSM COEFFICIENTS

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Plethysm coefficients are among the most mysterious structure constants in algebraic combinatorics, arising in the expansion of plethysms of Schur functions. Despite their importance in the representation theory of general linear groups and symmetric functions, a satisfactory combinatorial interpretation of these coefficients remains unknown. Consequently, various stability phenomena exhibited by plethysm coefficients have attracted significant attention.

In this talk, we present elementary combinatorial proofs of several stability results concerning the plethysm of complete symmetric functions. Our approach is based on a direct connection between plethysm coefficients and vector-partition functions. We first show that the monomial expansion of $h_n[h_m]$ can be interpreted in terms of vector partitions with prescribed numbers and sizes of parts. Using the Jacobi–Trudi identity, this yields an explicit expression for plethysm coefficients as alternating sums of vector-partition functions.

We then exploit simple stability properties of vector partitions to obtain short proofs of two important results. First, we prove that for any fixed partition λ, the sequence of coefficients $\langle h_n[h_m], s_{\lambda[mn]}\rangle$ stabilizes once $m, n \geq |\lambda|$, recovering a recent stability theorem of Bowman and Paget. As a consequence, one obtains the equality $\langle h_n[h_m], s_{\lambda[mn]}\rangle = \langle h_m[h_n], s_{\lambda[mn]}\rangle$ in the stable range. Second, we establish the eventual stability of the coefficients $\langle h_n[h_{m+d}], s_{\lambda+(nd)}\rangle$, providing a combinatorial proof of a theorem originally proved by Brion in connection with a conjecture of Foulkes.

These results demonstrate that key stability phenomena for plethysm coefficients arise naturally from elementary properties of vector partitions, yielding transparent combinatorial proofs that avoid more sophisticated geometric and representation-theoretic techniques.