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## PARKING TREES AND THE TORIC $g$ -VECTOR OF NESTOHEDRA

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In his 1987 paper on intersection cohomology and generalized  $h$ -vectors, Richard Stanley computed the toric  $g$ -vector of the permutahedron up to dimension 5 as a potential fruitful example for understanding this invariant. Twenty-five years later he described the toric  $g$ -vector of an Eulerian poset as “an exceedingly subtle invariant”. For the face lattice of rational convex polytopes, Stanley proved that the nonnegativity of the toric  $g$ -vector follows from the hard Lefschetz theorem for the intersection homology of projective toric varieties, and for general convex polytopes nonnegativity follows from Karu’s hard Lefschetz theorem for combinatorial intersection homology.

We give combinatorial interpretations of the toric  $g$ -vector of three classical polytopes (the associahedron, the cyclohedron and the permutahedron) in terms of 123-avoiding objects. Using work of Postnikov-Reiner-Williams, we extend our combinatorial model to all chordal nestohedra using the notion of parking trees. As a corollary we obtain combinatorial proofs of nonnegativity of the toric  $g$ -vector for all of these polytopes.

We end with open questions.

Joint work with Gábor Hetyei and Margaret Readdy.