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TWO-COLORS LATTICE PATHS WITH CATASTROPHES AND APPLICATIONS TO VACATION QUEUES

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We consider a class of two-colors weighted lattice paths motivated by discrete-time $M/M/1$ -type queues with vacations and disasters. The blue color corresponds to the vacation regime, and the red color corresponds to the working regime. A disaster sends the path from a positive level directly to level 0.

The main combinatorial feature is the transition from the blue regime to the red regime. This transition keeps the same level but changes the color, and therefore creates a decomposition structure that is not present in the usual one-color model with catastrophes. We derive bivariate generating functions for the path ending in each regime, using the kernel method. The kernel root is interpreted as a red first-passage generating function. We then use this interpretation to obtain the generating function of the busy cycle, explicit coefficient formulas via the ballot theorem, and asymptotic estimates for the cycle length.

We also indicate how the same approach extends to two-colors Motzkin paths, which arise from the uniformized version of the corresponding continuous-time queue.

Joint work with David Perry.