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ELECTRICAL NETWORKS, GRASSMANNIANS, AND CLUSTER ALGEBRAS

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Our talk is dedicated to the deep combinatorial and algebraic structures underlying the theory of electrical networks. An electrical network is a planar graph in which each edge is assigned a positive conductivity, and the vertices are divided into inner nodes and boundary nodes. The cornerstone object in electrical network theory is the response matrix, which linearly relates the voltages applied to the boundary nodes to the currents that flow through them. A classical theorem characterizes these matrices by three conditions: symmetry, zero row sums, and circular non-negativity – a certain sign condition on an exponentially large family of minors.

An important applied problem is to describe minimal tests for circular non-negativity: the minimal sets of circular minors whose positivity guarantees the positivity of all others. This problem can be reduced to a purely combinatorial problem involving the analysis of groves – spanning forests with boundary-connected components. The grove measurements L_σ, which are generating functions of groves with a fixed boundary, serve as natural projective coordinates on the space of electrical networks. These coordinates satisfy quadratic relations known as the electrical Plücker relations $\mathcal{I}_n$. This fact is equivalent to the existence of an embedding (the Lam embedding) of the space of electrical networks into $\mathrm{Gr}(n-1, 2n)$.

It is well known that the homogeneous coordinate ring of $\mathrm{Gr}(k, m)$ admits a cluster algebra structure. Moreover, each seed of this algebra, consisting entirely of Plücker coordinates, is the positivity test – that is, the positivity of all Plücker coordinates in the seed implies the positivity of all remaining coordinates. By carefully applying this general result to points of the Grassmannian $\mathrm{Gr}(n-1, 2n)$ lying in the image of the Lam embedding, one can obtain criteria for circular positivity.

Moreover, this technique not only provides a description of circular positivity tests for response matrices but also allows one to construct a cluster structure on the homogeneous coordinate ring of the Lagrangian Grassmannian $\mathrm{LG}(n-1, 2n-2)$, which is isomorphic to the grove algebra $\mathcal{G}_n = \mathbb{C}[L_\sigma]/\mathcal{I}_n$.

Joint work with Boris Bychkov and Lazar Guterman..